Graphs of Polar Curves

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Objectives of Assignment

1. To learn how X(PLORE) graphs polar curves.
2. To examine common polar functions and become familiar with their curves.

******WARNING******
Mastering the basics of polar coordinates requires four skills:

1. Translating rectangular coordinates into polar coordinates and vice-versa.
2. Translating rectangular expressions into polar expressions and vice-versa.
3. Graphing polar curves by plotting points.
4. Becoming familiar with polar functions and their graphs.

This lab unit is only designed to help you with #2 and #4. To do well on exams, in other classes, and in job assignments, you will need to master the other two by reading and working problems in your text.

I. Two Simple Requirements for Plotting Polar Curves With X(PLORE)

In order to make circles look like circles instead of ellipses, the graphing window must always have an x:y ratio of 4:3. That is, always use windows such as (–4, 4, –3, 3), (–8, 8, –6, 6), or (–12, 12, –9, 9). This compensates for the fact that the computer screen is not square. Also, type in and enter the command *grid(1, 1)* to provide a grid on your graph.

X(PLORE) uses the command "polarg(r(t), t = θ_1 to θ_2)" to plot polar graphs.

II. Circles

Type and enter the following four lines:

window(–4, 4, –3, 3)
grid(1, 1)
r(t) = 1
polarg(r(t), t = 0 to 2π)

Get back to the input screen, change "*r(t) = 1*" to "*r(t) = 2*", press ENTER, and then ENTER the "polarg" line to re-graph.

Then change to "*r(t) = 3*" and re-graph.

Change to "*r(t) = sin(t)*" and re-graph.
Change to "*r(t) = 2sin(t)*" and re-graph.
Change to "*r(t) = 3sin(t)*" and re-graph.

Change to "*r(t) = cos(t)*" and re-graph.
Change to "*r(t) = 2cos(t)*" and re-graph.
Change to "*r(t) = 3cos(t)*" and re-graph.

Change to "*r(t) = cos(t) + sin(t)*" and re-graph.
Change to "*r(t) = –2sin(t)*" and re-graph.
Change to "*r(t) = –3cos(t)*" and re-graph.

Each of these polar graphs appears to be a circle and, indeed, each can be shown to be a circle with a particular center and radius. For example, with *r(t) = cos(t) + sin(t),*

$r(t) = cos(t) + sin(t),$ or
$r = cos(t) + sin(t)$
$r^2 = rcos(t) + rsin(t)$ (multiplied both sides by r)
$x^2 + y^2 = rcos(t) + rsin(t)$ (since $r^2 = x^2 + y^2$)
$x^2 + y^2 = x + y$
$x^2 - x + y^2 - y = 0$
$(x - \tfrac{1}{2})^2 + (y - \tfrac{1}{2})^2 = \tfrac{1}{2}$ (by completing the square)

So, *r(t) = cos(t) + sin(t)* is equivalent to $(x - \tfrac{1}{2})^2 + (y - \tfrac{1}{2})^2 = \tfrac{1}{2}$. As you know, this is a circle centered at $(\tfrac{1}{2}, \tfrac{1}{2})$ with radius $\tfrac{1}{\sqrt{2}}$.

II. Cardioids and Limaçons

Now, to erase all the circles, type "erase" on a separate line and press ENTER.

Change to "*r(t) = cos(t)*" and graph. (This is a circle.)
Without erasing, change to "*r(t) = cos(t) – 0.25*" and re-graph. (This is a limaçon.)
Without erasing, change to "*r(t) = cos(t) – 0.5*" and re-graph. (This is a limaçon.)
Without erasing, change to "*r(t) = cos(t) – 0.75*" and re-graph. (This is a limaçon.)
Without erasing, change to "*r(t) = cos(t) –1*" and re-graph. (This is a cardioid.)
Without erasing, change to "*r(t) = cos(t) – 1.25*" and re-graph. (This is a limaçon.)

Erase. Now change to "*polarg(r(t), t = 0 to π)*" – that is, have t go from 0 to π instead of to 2π. Change to "*r(t) = cos(t) – 0.5*" and graph.

Without erasing, change to "*r(t) = cos(t) –1*" and re-graph.

Without erasing, change to "*r(t) = cos(t) –1.25*" and re-graph.

It is clear that only half of the limaçons and cardioid are being graphed.

Without erasing, change to "*r(t) = cos(t)*" and re-graph. Why is the whole circle graphed instead of only half of it?

Limaçons and cardioids can be created using sines instead of cosines, and their sizes can be altered.

III. n-Petaled Roses

Erase. Change back to "*polarg(r(t), t = 0 to 2π)*".

Change to "*r(t) = 2sin(3t)*" and graph. Count the petals.
Without erasing, change to "*r(t) = 2sin(5t)*" and re-graph. Count the petals.
Without erasing, change to "*r(t) = 2cos(3t)*" and re-graph. Count the petals.
Without erasing, change to "*r(t) = 2cos(5t)*" and re-graph. Count the petals.
Notice any pattern in the number of petals?

Erase.

Change to "*r(t) = 3sin(2t)*" and graph. So much for that pattern.
Without erasing, change to "*r(t) = 3sin(4t)*" and re-graph.
Without erasing, change to "*r(t) = 3cos(2t)*" and re-graph.
Without erasing, change to "*r(t) = 3cos(4t)*" and re-graph.
Any ideas on how to make a 6-petaled rose?

IV. Other Familiar and Unfamiliar Curves

Lines: Erase. Change to:
window(–8, 8, –6, 6)
grid(2, 2)
r(t) = sec(t) and graph.

Without erasing, change to "*r(t) = 3sec(t)*" and re-graph.
Without erasing, change to "*r(t) = 4sec(t)*" and re-graph.
Without erasing, change to "*r(t) = 3csc(4t)*" and re-graph.

Spirals: Erase. Change to "*r(t) = t*" and graph.
Without erasing, change to "*r(t) = 0.5t*" and re-graph.

Without erasing, change to “*r(t) = sqrt(t)*” and re-graph.
Without erasing, change to “*r(t) = e^t*” and re-graph (use ALT E to get *e*).

Conchoids: Erase. Change back to “*grid(1, 1)*”.
Change to “*r(t) = 3 – 1.5sec(t)*” and graph.
Change to “*r(t) = 2 – csc(t)*” and graph.

Cissoid of Diocles: Erase. Change to “*r(t) = sin(t)tan(t)*” and graph.

Conic Sections: Erase. Type and ENTER another line “*a = 0.5*”.
Change to “r(t) = a/(1 – a*cos(t)) and graph. An ellipse.
Change to “*a = 1*” and re-graph. A parabola.
Change to “*a = 1.5*” and re-graph. A hyperbola.
All three of these conic sections have a focus at the same point.